Faster Determination of the Intraseasonal Variability of Storm Tracks Using Murakami’s Recursive Filter

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ABSTRACT

Murakami’s recursive filter technique is suitable for computing storm tracks with reduced needs in data length, and it decreases computing time by the factor 3.5. It is shown that the storm tracks differ only slightly from the ones obtained using the conventional Blackmon filtering approach. A problem identified with respect to the exact frequency of the 1.0 response of Murakami’s filter appears to be of minor importance.

1. Introduction

The midlatitude storm tracks have been defined by Blackmon (1976) as regions of maximum standard deviations of geopotential height with respect to periods between 2.5 and 6 days. In the Northern Hemisphere, the storm tracks are subdivided into two quasi-zonally oriented areas over the Atlantic and the Pacific (cf., Blackmon 1976; Lau and Wallace 1979; Wallace et al. 1988; Lau 1988). Apart from a mean annual cycle in their location, extension, and intensity, they undergo considerable variability on both the interannual and the intraseasonal timescales. For example, Nakamura (1992) showed in his Fig. 3 that the time mean activity of the Pacific storm track is suppressed in midwinter. There is no doubt that this must be the result of very different developments in individual winters. Efforts toward an understanding of this variability will require a large number of storm track investigations on short time intervals. It is thus important to have an efficient and convenient procedure for isolating the storm track variability from the total spectrum.

Bandpass filtering can be done by using recursive or nonrecursive filtering. We consider here the recursive filter described in the appendix of Murakami (1979) and compare its performance for computing storm tracks with the conventional Blackmon filter (Blackmon 1976; Blackmon and Lau 1980). Murakami designed the following “second-order” bandpass filter whose output is obtained by

\[ y_k = a(x_k - x_{k-2}) - b_1 y_{k-1} - b_2 y_{k-2}. \]  

Implementing forward and reverse recursion filtering (i.e., the output is reversed in time and processed again for final output) to obtain zero phase shift for all frequencies leads to the following response function:

\[ R(\omega) = W(Z) W^*(Z), \]

with

\[ W(Z) = \frac{a(1 - Z^2)}{1 + b_1 Z + b_2 Z^2}, \quad Z(\omega) = e^{-\omega \Delta t}. \]

Here \( W^*(Z) \) denotes the complex conjugate of \( W(Z) \), and coefficients \( a \), \( b_1 \), and \( b_2 \) are functions of \( \omega_1 \) and \( \omega_2 \) denoting the lower and upper cutoff frequency (0.5 filter response), respectively. The corresponding amplitude response function is plotted on the left side of Fig. 1. For our purposes, we designed the filter to be somewhat broader at the lower frequency end than that given in the above definition of storm tracks, and we set \( \omega_1 = 2\pi/8.0 \text{ day}^{-1} \) and \( \omega_2 = 2\pi/2.5 \text{ day}^{-1} \). As will be shown later, this ensures that the maximum of the amplitude response function is close to a period of 4 days, where a relative maximum in the power spectrum of the 500-hPa geopotential height is observed (Fraedrich and Böttger 1978).

In his original work, Blackmon (1976) used a nonrecursive “medium pass” filter (periods 2.5 < \( T < 6 \) days), whose frequency responses and coefficients for twice-daily datasets are given in that work. Since we are using daily data, the 21-point filter developed by Blackmon and Lau (1980) is considered here. Its amplitude response function is plotted on the right side of Fig. 1. The main characteristics of this medium-pass filter are its relatively sharp cutoff frequencies and broad pass band. These features are desirable for many purposes. A disadvantage of this filter is the phenomenon of Gibbs oscillations, that is, overswaying (response greater than 1.0) in the pass band.

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Fig. 1. Amplitude response function $R(f)$. (Left) Murakami's bandpass filter with cutoff frequencies $\omega_1 = 2\pi/8.0$ day$^{-1}$ and $\omega_2 = 2\pi/2.5$ day$^{-1}$. (Right) Blackmon's medium-pass filter with cutoff frequencies $f_1 \approx 1/5.9$ day$^{-1}$ and $f_2 \approx 1/2.6$ day$^{-1}$.

2. Results

An intercomparison of the two filtering methods reveals two outstanding advantages of Murakami's technique. First the number of floating-point operations is about 3.5 times smaller than with Blackmon's procedure due to the low-order recursion formula implemented. As a consequence, the total computing time is reduced by about the same factor. Second, there is no reduction in the length of the filtered time series com-

Fig. 2. Pacific storm tracks in 500 hPa for 6–20 January (left column) and for 21 January–4 February 1990 (right column) with Blackmon filter (upper row), Murakami filter (35 days filtered and 15 days extracted—middle row), and Murakami filter (15 days filtered—lower row). Contour interval is 10 gpm; areas over 60 gpm are hatched.
three days, as measured by the impulse response function (not shown), which is the inverse Fourier transform of the amplitude response function. This high rate of convergence is fundamental for the excellent performance of the filter, as will be shown further below.

A comparison of the results obtained with the two filter techniques is carried out for the Pacific storm track. Both the structure and the intensity of the storm track are computed for two adjacent 15-day time intervals (6–20 January and 21 January–4 February 1990) by using analyses from the European Centre for Medium-Range Weather Forecasts (ECMWF) that have a resolution of 2.5° latitude by 2.5° longitude. The Blackmon filter and the Murakami filter are applied to time series of 500-hPa geopotential height from which the mean value and the trend has been removed. Subsequently, the standard deviation is calculated at each grid point. As mentioned before, the input dataset for Blackmon filtering requires a minimum extension of 10 days at each end; thus, we require an input time series of 35 days for each case.

The resulting storm track structures and positions after application of the Blackmon filter are shown in the upper row of Fig. 2. A clear change in storm track intensity and structure between the two cases is evident. The same two 35-day time series were also processed with Murakami’s method, and the corresponding 15-day periods were extracted. Storm tracks computed with this method are shown in the middle row of Fig. 2. Comparison of the results obtained with each filter shows that the position and extension of the Pacific storm track coincides very well in each time interval. The maximum of variability is about 6% lower with Murakami filtering; but the variability is also higher at some locations. Such differences are not unexpected because of the different filter characteristics.

The advantage of the recursive filter with respect to reduced data needs is investigated by filtering the pure 15-day intervals separately. The storm tracks obtained (see bottom panels of Fig. 2) deviate less than 3% from those in the middle panel, which further emphasizes the fast convergence of the filter. Thus, even for very short input, one can justify a filtering procedure based on the input time series under investigation. It is clear, however, that a reasonable relationship between the lower cutoff frequency and the minimum length of the time series is required. In our case, no intervals shorter than 15 days should be considered.

When the climatological mean is considered, the Murakami filter in general retains more variability than the Blackmon filter, while the field structures obtained with both techniques coincide well. The former effect is assigned to the flatter cutoff slopes in the amplitude response function. As an example, 10-yr mean fields (1980–89) of January storm tracks in the Northern Hemisphere are shown in Fig. 3. The relative deviation between the mean fields is about 3% in the storm track centers and up to 15% elsewhere in the midlatitudes.

Fig. 3. Climatological distribution of 500-hPa Northern Hemisphere storm tracks in January (1980–89, ECMWF data) obtained with the Blackmon filter (top) and with the Murakami filter (bottom). Contour interval is 10 gpm; areas over 60 gpm are hatched.
The good agreement of local storm track structures obtained with both filters was confirmed when considering the Southern Hemisphere, other levels, and other parameters (e.g., wind and eddy kinetic energy). In those regions where the spectral energy distribution of the quantity considered does not match the filter characteristics designed for storm track identification, the relative differences become larger. The investigation of the variability features prevailing in these areas requires that the filter characteristics are adapted. For example, Sperling (1993) successfully applied the Murakami filter to the tropical Madden–Julian oscillation (period range 30–60 days).

While there are clear advantages of Murakami’s recursive filter, there is also a problem that should be noted. According to Murakami, the frequency \( \omega_0 \) that results in a 1.0 response is provided by the relation

\[
\omega_0^2 = \omega_1 \omega_2. \tag{4}
\]

We found, however, that this relation is an approximation that holds true only in the low-frequency regime \( (\omega_1 \omega_2 \geq 4\pi^2/23.0 \text{ day}^{-2}) \) together with a narrow bandwidth \( (\omega_2 - \omega_1 \leq 2\pi/10.0 \text{ day}^{-1}) \). While in our case 0.5 response is obtained at \( \omega_1 = 2\pi/8.0 \text{ day}^{-1} \) and \( \omega_2 = 2\pi/2.5 \text{ day}^{-1} \), there is a clear shift of the function’s maximum from the value \( \omega_0 = 2\pi/4.47 \text{ day}^{-1} \) expected from (4) to the observed \( \omega_0^* = 2\pi/3.71 \text{ day}^{-1} \). This discrepancy was present in Murakami’s original work, but it was not evident because of the different range of frequencies selected there. Murakami chose the frequencies \( \omega_1 = 2\pi/6.0 \text{ day}^{-1} \) and \( \omega_2 = 2\pi/3.75 \text{ day}^{-1} \). This yields \( \omega_0^* = 2\pi/4.65 \) for 1.0 response, which differs from his value \( \omega_0 = 2\pi/4.74 \) according to (4). Inserting \( \omega_0 = 2\pi/4.74 \) in (2) gives a 0.9937 response. Shifting the filter into the high- or low-frequency regime makes it clear that the reduced response cannot be due to round-off errors since the deviations from (4) become larger or smaller, respectively.

The importance of this shift in maximum response frequency is difficult to quantify, as \( \omega_0 \) is a function of \( \omega_1 \) and \( \omega_2 \) and cannot be changed without changing the cutoff frequencies as well. In light of the results shown in Fig. 2, the influence of the deviation between \( \omega_0^* \) and \( \omega_0 \) appears to be small for the applications suggested in this work.

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**REFERENCES**


